

Interest Rate Uncertainty and Sovereign Default Risk*

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PRELIMINARY AND INCOMPLETE

Abstract

As the United States emerged from the Great Recession, there was considerable uncertainty around the future direction of US monetary policy exemplified by the chatter and speculation around tapering of quantitative easing by the US Fed in the financial press. The increased uncertainty around the timing and speed of the tapering coincided with a sharp spike in the sovereign bond yields of several emerging economies. We explore the impact of an increase in interest rate uncertainty on the borrowing costs of a small open economy in an otherwise standard model of sovereign default, where spread is endogenous. We find that introducing time-varying volatility in the world interest rate (i.e. uncertainty shocks) the model predicts a mean sovereign spread that is 115% larger and 126% more volatile. The model also predicts that countries default more than twice as frequently. Moreover, the equilibrium debt-to-income ratio is 19% lower. The welfare gains from eliminating uncertainty about the world interest rate amount up to a 1.8% permanent increase in consumption. Overall, we find quantitative support for the widespread concerns regarding the uncertainty about when and how the Fed will unwind its quantitative easing.

JEL classification: E32, F34, F41.

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1 Introduction

The behavior of the T-bill rate is always in the watch-list of policy makers and investors, both in advanced and developing countries. As the United States emerged from the recent Great Recession, there was considerable uncertainty around the future direction of US monetary policy as well as much speculation about when and how will the US Fed unwind its quantitative easing (QE) program. For instance, the reaction of global markets (particularly in emerging market economies, EMEs) after the summer of 2013’s “tapering talk” was uncommon and different from the usual market response to Fed monetary policy actions.¹ A sharp market adjustment followed in EMEs, including a reversal in capital flows, and a spike in government bond yields (see figure 1). EMEs perceived this “tapering talk” as a sign of earlier than anticipated tightening of U.S. monetary policy and reacted in that way. On average, sovereign spreads across EMEs rose by 1%, currencies depreciated by 3%, and equities fell by 7%.

That uncertainty has not gone away and, as it is usually the case, if the markets care about it so do policy makers. Throughout the emerging world voices have recently been raised calling for a decision to be made about the future of the US monetary policy and urging the Federal Reserve to put an end to the uncertainty. The following quotes (taken from a recent article on the Financial Times) summarize these concerns:²

“We think US monetary policymakers have got confused about what to do. The uncertainty has created the turmoil.”

Mirza Adityaswara, Senior Deputy Governor, Indonesia Central Bank.

“The uncertainty about when the Fed hike will happen is causing more damage than the Fed hike will itself.”

Julio Velarde, Governor, Peru Central Bank.

Motivated by these facts and policy concerns, we develop an equilibrium model of sovereign default to study the relationship between endogenous country spreads and world interest rate

¹ In his May 22, 2013 testimony to Congress, Fed chairman Ben Bernanke suggested the possibility of a tapering (i.e. a reduction in bond purchases by the Fed). This testimony together with the release of FOMC minutes triggered a global reassessment of expectations around the timing and path of adjustment in U.S. monetary policy. See Mishra et al. (2014).

²Financial Times, September 9, 2015.

uncertainty. To do so, we introduce stochastic volatility into the world interest rate process (as modeled by Fernández-Villaverde et al., 2011) in an otherwise standard quantitative model of sovereign debt and default that produces an endogenous sovereign spread, in the Eaton and Gersovitz (1981) tradition. Since debt contracts are not enforceable in the model, defaults can occur in equilibrium and the spread charged to the sovereign captures this probability of default. We analyze the role that shocks to both the level and the volatility of the world interest rate play in explaining the long run dynamics of the sovereign spread and the default risk of emerging economies. In particular, we explore the impact of an unexpected increase in interest rate uncertainty on the macroeconomic dynamics of a small open economy (SOE).

We make three contributions. First, we develop a general equilibrium model of sovereign debt with endogenous default and endogenous country spreads wherein investors face a stochastic world interest rate rather than a constant one, which provides a more accurate representation of the market conditions. Our framework is able to quantify the impact of such shocks and inform the policy discussion about the effects of uncertain unwinding of the Fed’s quantitative easing. Second, this paper provides a mechanism by which changes in world interest rate *uncertainty* could affect the sovereign default risk, a country’s borrowing decisions, and the sovereign bond spread even when the *level* of the interest rate itself is fixed. Third, we show how an unexpected increase in world interest rate uncertainty can generate macroeconomic dynamics as the ones observed in EMEs after the tapering talk the summer of 2013.

A version of our model, calibrated to Argentina, generates the following main findings: (i) introducing uncertainty about world interest rate fluctuations (measured by both interest level and volatility shocks) more than doubles the default risk, increases the mean spread by 115% and the volatility of the spread by a 126%; (ii) in response to uncertain world interest rates the country optimally decides to lower its “exposure” and decrease the average debt-to-income ratio from 60% to 49%; (iii) the volatility of the world interest rate alone is responsible for in between 24% and 39% of the observed variations; (iv) we show that the welfare gains of eliminating world interest rate uncertainty are sizable, amounting up to a 1.8% permanent increase in consumption; and (v) the model also generates Impulse Response Functions in response to an unexpected increase in world interest rate uncertainty that are consistent with the observed behavior of EMEs during the ‘tapering talk’ of 2013. Moreover, the model matches well the other business cycle moments observed in the data.

The rest of the paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 describes the model and defines the equilibrium. Section 4 discusses the numerical solution and the calibration. Section 5 presents the results and section 6 concludes. All the tables and figures are in the appendices.

2 Related Literature

There is ample evidence that movements in the international risk-free rate (i.e. the T-bill rate) have macroeconomic consequences for emerging economies. Neumeyer and Perri (2005) report that real country interest rates in emerging economies are strongly countercyclical and tend to lead the cycle. They also find that an exogenous interest rate shock can account for up to 50 percent of the volatility of output in Argentina. Uribe and Yue (2006) report that a strong relation exists among the world interest rate, the country spread and emerging market fundamentals. In particular, they show that US interest rate shocks and country spread shocks can explain the large movements in aggregate activity in emerging economies. García-Cicco et al. (2010) also find that the country spread shock is one of the most important drivers of emerging economies business cycles. They show that an exogenous country spread shock and a preference shock can explain a large fraction of aggregate fluctuations in the Argentine business cycles. All these papers take the country spread as an exogenous variable with a time-invariant volatility, while our work endogenizes the spread (as a result of defaulting incentives on the side of the sovereign) and allows for it to have a time-variant volatility.

Fernández-Villaverde et al. (2011) study the impact of time-varying volatility on the macroeconomic dynamics of a SOE. They examine the effects on the business cycles of four emerging economies - Argentina, Ecuador, Venezuela and Brazil. We follow Fernández-Villaverde et al. (2011) in the approach to modeling the stochastic behavior of the world interest rate, while departing from their approach to modeling the country spread: as already noted above, our model is one of endogenous spreads. Then, we explore the mechanism by which world interest rate uncertainty affects the country spread and default risk in emerging economies. We see our work to be complementary to the analyzes in Neumeyer and Perri (2005), Uribe and Yue (2006), García-Cicco et al. (2010), and Fernández-Villaverde et al. (2011).

Within the quantitative literature on sovereign defaults (following Eaton and Gersovitz,

1981, Aguiar and Gopinath, 2006, and Arellano, 2008) our paper is particularly related to two other studies. The first one is by Seoane (2014). He studies how changes in the aggregate income volatility affect the sovereign spreads of four European economies: Greece, Italy, Portugal and Spain. He presents a model in the spirit of Arellano (2008) and incorporates time-varying volatility of the income process which generates substantial variability in spreads. Our work complements his by keeping the income process with a time-invariant volatility and putting the time-varying volatility in the world interest rate process. We share results on the precautionary savings motive that makes sovereigns borrow less when facing a more uncertain environment. The second paper in the default literature to which our work relates is the one by Pouzo and Presno (2012). These authors study the problem of a SOE that can default on its obligations, under model uncertainty: lenders fear that the probability model of the underlying state of the borrowing economy is misspecified and hence may demand higher returns on their investments. Even though our paper tackles a different type of uncertainty (i.e. time-varying volatility of the world interest rate) the results are consistent: more uncertainty leads to higher and more volatile spreads while maintaining historically low default rates.

Finally, this paper is also related to the literature on uncertainty shocks. For instance, Justiniano and Primiceri (2008) and Bloom (2009) study the effect of changes in the volatility of technology shocks in general equilibrium models for closed economies. Justiniano and Primiceri (2008) study the changes in volatility in postwar US data by estimation of a large-scale dynamic stochastic general equilibrium model (DSGE) allowing for time variation in the structural innovations. They find that shocks specific to investment are mostly responsible for the observed “great moderation.” Bloom (2009), on the other hand, shows that uncertainty shocks can generate short sharp recessions and recoveries.

3 Model

Our environment follows closely the model in Arellano (2008). We study a real model of a SOE that trades one-period non-contingent bonds with a large pool of international investors. Bond contracts are not enforceable. Time is discrete and goes on forever.

3.1 Households

The economy is populated by identical households. They rank consumption streams according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1)$$

where $0 < \beta < 1$ is the subjective discount factor, c denotes consumption, and $u(\cdot)$ is a period utility function which satisfies $u' > 0$, $u'' < 0$.

Households receive an exogenous stream of income which follows a log-normal AR(1) process:

$$\log(y_t) = \rho_y \log(y_{t-1}) + \varepsilon_{y,t} \quad (2)$$

where ε_y is an *i.i.d.* $N(0, \sigma_\varepsilon^2)$.

The government has access to the international financial market where it trades one-period non-contingent bonds, b_{t+1} , with risk neutral competitive foreign investors at a price q_t . Given that bond contracts are not enforceable, the government can decide whether to repay its debts. Consequently, every period t the government is in one of two states: default or non-default. If the government chooses to repay its outstanding debt, b_t , then it has access to the international credit market and the resource constraint of the economy is given by:

$$c_t = y_t + b_t - q_t b_{t+1} \quad (3)$$

On the other hand if the government declares a default, it remains in default for a stochastic number of periods. While the government is in default, it cannot issue debt and domestic aggregate income is reduced by $\phi(y)$. As in Arellano (2008) and Chatterjee and Eyigungor (2012), we assume that it is proportionally more costly to default in good times ($\phi(y)/y$ is increasing in y).³ The resource constrain for the default case can then be written as:

$$c_t = y_t - \phi(y_t) \quad (4)$$

³Arellano (2008) and Chatterjee and Eyigungor (2012) show that this property is important in accounting for the dynamics of the sovereign debt interest rate spread. Mendoza and Yue (2012) show that this property of the cost of defaulting arises endogenously in a setup in which defaults affect the ability of local firms to acquire a foreign intermediate input good. Sosa-Padilla (2015) shows that a model of a sovereign defaulting on its own financial sector can generate endogenous default costs that share this property.

3.2 Foreign Investors

There is a large number of risk neutral international investors who trade bonds with the domestic economy. These lenders have deep pockets and face an opportunity cost of funds given by a time-varying world risk free interest rate, r_t . They maximize expected profits taking prices as given, that is:

$$\max_{b_{t+1}} \Pi_t = q_t b_{t+1} - \frac{1 - \delta_{t,t+1}}{1 + r_t} b_{t+1} \quad (5)$$

where $\delta_{t,t+1}$ is the probability of default in period $t + 1$, as of period t . The first order condition implies a bond price function as follows:

$$q_t = \frac{1 - \delta_{t,t+1}}{1 + r_t} \quad (6)$$

Equation (6) is the pricing equation found in most of the sovereign default literature. It states that risk-neutral investors will price bonds as the discounted repayment probability.

3.3 Law of Motion for the World Interest Rate

Following Fernández-Villaverde et al. (2011) we specify the international risk-free rate faced by investors as:

$$r_t = \bar{r} + \varepsilon_{r,t} \quad (7)$$

where \bar{r} is the mean of world risk-free real rate, and $\varepsilon_{r,t}$ represents deviations from this mean. In particular, we assume the following AR(1) behavior for $\varepsilon_{r,t}$:

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t} \quad (8)$$

where $u_{r,t}$ is a normally distributed shock with mean zero and unit variance. The crucial ingredient in this stochastic process is that the standard deviation ($\sigma_{r,t}$) is not constant but time-varying and itself follows another (independent) AR(1) process:

$$\sigma_{r,t} = (1 - \rho_{\sigma_r}) \bar{\sigma}_r + \rho_{\sigma_r} \sigma_{r,t-1} + \eta_r u_{\sigma_r,t} \quad (9)$$

where $u_{\sigma_r,t}$ is another normally distributed with mean zero and unit variance. We further assume that $u_{r,t}$ and $u_{\sigma_r,t}$ are independent from each other. The parameters $\bar{\sigma}_r$ and η_r measure the degree of mean volatility and stochastic volatility in the international risk free rate. A high $\bar{\sigma}_r$ corresponds to a high mean volatility and a high η_r corresponds to a high degree of stochastic volatility in the international risk free rate.

3.4 Timing

The timing of events, for a government that is not excluded from financial markets, is as follows. The government starts with an initial bond position b_t and observes the realizations of the income level (y_t), the world interest rate level (r_t) and the interest rate volatility shock ($\sigma_{r,t}$), and then decides whether to repay its outstanding debt. If it decides to repay, it chooses b_{t+1} subject to the resource constraint, taking the bond price schedule $q_t(b_{t+1}; y_t, r_t, \sigma_{r,t})$ as given. Finally, consumption takes place.

On the other hand, if government decides to default it gets excluded from financial markets and suffers a direct income loss. In case of default, there is no other decision to be made as the level of consumption equals the (reduced) income level. At the end of the period, a re-access coin is tossed, and the government will re-access (remain excluded from) financial markets in the following period with probability μ ($1 - \mu$).

3.5 Recursive Equilibrium

We now turn to recursive notation, where *primes* denote next-period value of the variables. Let $\Theta = \{y, r, \sigma_r\}$ denote the aggregate exogenous state. Given a next-period bond position b' and a realization of Θ , the price of a bond satisfies:

$$q(b'(b; \Theta); \Theta) = \frac{1 - \delta(b'(b; \Theta); \Theta)}{1 + r} \quad (10)$$

The optimal default decision is taken as:

$$v^0(b; \Theta) = \max_{d \in \{0,1\}} \{(1 - d)v^c(b; \Theta) + dv^d(\Theta)\} \quad (11)$$

where d equals 1 (0) if the government chooses to (not to) default. Under no-default, the government solves the following problem:

$$v^c(b; \Theta) = \max_{b'} \{u(y + b - q(b'; \Theta)b') + \beta \mathbb{E}_{\Theta'} [v^0(b'; \Theta') \mid \Theta]\} \quad (12)$$

Under default, the value function is given by:

$$v^d(\Theta) = u(y - \phi(y)) + \beta \mathbb{E}_{\Theta'} [\mu v^0(0; \Theta') + (1 - \mu)v^d(\Theta') \mid \Theta] \quad (13)$$

where, in order to keep the environment as simple as possible, we assume that when the government gains re-access to financial markets it does so with no debt obligations (i.e. it gets a “fresh start”).⁴

The government default policy can then be characterized by repayment sets, $A(b)$, and default sets, $D(b)$, for a given level of assets b as follows:

$$A(b) = \{\Theta \in \mathbb{Y} \times \mathbb{R} \times \Sigma : v^c(b; \Theta) \geq v^d(\Theta)\} \quad (14)$$

$$D(b) = \{\Theta \in \mathbb{Y} \times \mathbb{R} \times \Sigma : v^c(b; \Theta) < v^d(\Theta)\} \quad (15)$$

where \mathbb{Y} , \mathbb{R} , and Σ are the sets of possible realizations for y , r and σ_r , respectively. Next, we define the recursive equilibrium of this economy.

Definition 1 *The recursive equilibrium for this economy is a set of policy functions for (i) consumption $c(b; \Theta)$; (ii) government’s asset holdings $b'(b; \Theta)$, repayment sets $A(b)$, and default sets $D(b)$; and (iii) the price function for bonds $q(b', \Theta)$ such that:*

1. *Households’ consumption $c(b; \Theta)$ satisfies the resource constraint, taking the government policies as given.*
2. *The government’s policy functions $b'(b; \Theta)$, repayment sets $A(b)$, and default sets $D(b)$ satisfy the government optimization problem, taking the bond price function $q(b', \Theta)$ as given.*
3. *Bonds prices $q(b', \Theta)$ reflect the government’s default probabilities and satisfy creditors’ expected zero profits.*

⁴ For studies with positive recovery rates and renegotiation between sovereigns and lenders see for example Yue (2010), D’Erasmus (2011), and Hatchondo et al. (Forthcoming).

The equilibrium bond price must satisfy the government’s optimization problem and the lenders’ expected zero profit condition, such that bond prices reflect the default probabilities. The default probabilities $\delta(b'(b; \Theta); \Theta)$ and the default set $D(b')$ are then related as follows:

$$\delta(b'(b; \Theta); \Theta) = \mathbb{E}_{\Theta'} \{ \mathbb{1}_{\Theta' \in D(b')} \} \quad (16)$$

where $\mathbb{1}_x$ is an indicator function that takes the value of 1 if x is true, and 0 otherwise.

4 Numerical Solution

We solve the model numerically using value function iteration with a discrete state space.⁵ We focus on Markov-perfect equilibria. We use Tauchen (1986)’s method to discretize the income shock, the interest level shock and the interest rate volatility shock.⁶ We solve for the equilibrium of the finite-horizon version of our economy, and we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium objects as the infinite-horizon economy equilibrium objects.

The functional form for the period utility is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad (17)$$

where γ is the coefficient of relative risk aversion. As in Chatterjee and Eyigungor (2012), we assume a quadratic loss function for income during a default episode:

$$\phi(y) = \max\{0, d_0 y + d_1 y^2\} \quad (18)$$

As explained by Chatterjee and Eyigungor (2012), this functional form for the income loss $\phi(y)$ is flexible enough to accommodate many cases. If $d_0 > 0$ and $d_1 = 0$, then the cost is

⁵The algorithm computes and iterates on two value functions: v^0 and v^d . Convergence in the equilibrium price function $q(\cdot)$ is also assured.

⁶ We use grids of evenly distributed points. For the endogenous state variable (b), we use a grid of size N_b . We then follow Seoane (2014)’s strategy to discretize the exogenous state space. We begin by creating a grid of size N_y for the income process. We then discretize the space for the world interest rate volatility shocks, σ_r , creating a grid of size N_{σ_r} . Finally, in order to discretize the space for the interest rate level we need to create a grid (of size N_r) for *each* possible level of the interest rate volatility, $\sigma_r(i)$, $i = 1, \dots, N_{\sigma_r}$. So, in effect we have a matrix of possible values of interest rate levels (of size $N_{\sigma_r} \times N_r$). The results presented in Section 5 are obtained using $N_b = 150$, $N_y = 300$, $N_{\sigma_r} = N_r = 10$.

proportional to income; if $d_0 = 0$ and $d_1 > 0$, then the cost increases more than proportionately with income; if $d_0 < 0$ and $d_1 > 0$, then the cost is zero in a region ($0 < y < -d_0/d_1$) and then increases faster than income (for $y > -d_0/d_1$). This last case is very similar to Arellano (2008)'s cost-of-default function.

4.1 Calibration

We define our baseline economy as one in which there are no interest rate level shocks (i.e. $u_r = 0$) nor interest rate volatility shocks (i.e. $u_{\sigma_r} = 0$). This baseline economy is calibrated to a quarterly frequency using data for Argentina from the period 1983.Q4 - 2001.Q4. Table 1 summarizes the parameter values.

We estimate equation (2) using quarterly real GDP per capita for Argentina ranging from 1983.Q4 till 2001.Q4. The data counterpart of $\log(y)$ is the deviation of the natural logarithm of GDP per capita from its trend (computed using HP-filter, with smoothing parameter 1,600). The re-entry probability μ is set to 0.0385 according to Chatterjee and Eyigungor (2012), which implies an average period of 6.5 years of financial exclusion.⁷

We assume that the representative agent in the sovereign economy has a coefficient of relative risk aversion γ of 2 and a discount factor β of 0.95. The average risk free rate (\bar{r}) is 1 percent. Those values are within the range of accepted values in studies of business cycles in small open economies.⁸

We are left with two parameters (the coefficients of the default cost function, d_0 and d_1) to assign values to. We calibrate these two parameters to match an annual default frequency of 3% and an average debt-to-income ratio of 60%. Argentina has defaulted 3 times on its external debt in the last 100 years, giving rise to our targeted default frequency. With respect to the indebtedness statistics, Cowan et al. (2006) report a debt-to-output ratio of 59.87% for the period 1990–2004.

Table 2 presents the parametrization of the stochastic processes that govern the behavior of

⁷Benjamin and Wright (2008) report Argentina as being in a state of default between 1982 and 1992 and between 2001 and 2005. The average exclusion period is 7.5 years by these measures. Gelos et al. (2011) report an average exclusion of 4 years for emerging economies.

⁸The value for the subjective discount factor may appear low for typical business cycle models. However it is relatively large for the quantitative sovereign default literature: for example Yue (2010) uses .7, Aguiar and Gopinath (2006) use .8, and Arellano (2008) uses .953.

the world interest rate. All the values in Table 2 come from Fernández-Villaverde et al. (2011).⁹ These authors estimate (the equivalent to) equations (8) and (9) using a likelihood-based approach. Parameter values reported correspond to the median of the posterior estimates.¹⁰ It is important to note that none of the parameters in Table 2 are relevant for the computation of our baseline economy (where there are no interest level nor volatility shocks, i.e. $u_r = u_{\sigma_r} = 0$); they only affect the quantitative performance of what we later on define as the “full model”, where all shocks are present.

5 Results

In this section we present the main results of our paper. Firstly, we show the ability of the baseline model to account for salient features of business cycle dynamics in Argentina. Secondly, we study the effect of introducing volatility shocks: in particular, we see how policy functions, default incentives and the overall quantitative performance of the model change. Thirdly, we present results for an ‘intermediate’ version of our model, where only level (but not volatility) shocks affect the world interest rate. Fourthly, we present a measure of the welfare cost of interest rate volatility. Fifthly, using Impulse Response Functions we show how the model economy responds to unanticipated shocks in the volatility of the world interest rate.

5.1 Performance of the Baseline Economy

Table 3 reports moments in the data and in our simulations of the baseline economy (as well as in our ‘full model’ which is described below). As in previous studies, we report results for pre-default simulation samples. The only exception is the default rate, which we compute using all simulation periods. We simulate the model for 1,500 samples of 3,000 periods each. We then discard the initial 1,500 periods as a burn-in and from the remaining periods we extract 1,000 samples of 32 consecutive quarters before a default.

⁹ Fernández-Villaverde et al. (2011) estimate equations (8) and (9) using monthly data for the T-bill rate. Their posterior estimates imply annualized average standard deviations for the risk-free interest rate of 38 basis points (with only mean volatility) and 44 basis points with both mean and stochastic volatility. We adjust their estimates of mean and stochastic volatility (η and $\bar{\sigma}_r$) so that our quarterly model produces the same average standard deviations in annualized terms. We keep the persistence of both shocks unchanged.

¹⁰ For more details on the estimation of the stochastic process of the world interest rate, see Fernández-Villaverde et al. (2011) and their online appendix.

The moments reported in Table 3 are chosen to illustrate the ability of the model to replicate distinctive business cycle properties of economies with sovereign risk. The third column of the table shows that the baseline economy approximates well the moments used as targets (the default frequency and the debt-to-income ratio) and it is broadly consistent with non-targeted moments in the data: consumption is more volatile than income, the trade balance is countercyclical and the sovereign spread is also countercyclical (as is often the case in economies facing sovereign risk, see Neumeyer and Perri, 2005 and Uribe and Yue, 2006).

Two moments of the data proved particularly difficult to account for (by the baseline model): the mean and the volatility of the sovereign spread. The baseline economy can only produce 2% of the observed spread and 9% of the observed volatility of the spread. It is by now well understood that a standard model of sovereign default (in the tradition of Eaton and Gersovitz, 1981) with one-period debt and risk-neutral lenders cannot produce spread levels and volatilities in line with the data while simultaneously matching the observed debt-to-income level and a historically low default frequency.¹¹ It is important to highlight that these two moments are explicitly un-targeted: we are specially interested in understanding how shocks to both the level and the volatility of the world interest rate affect the mean and volatility of the spreads paid by a sovereign borrower under risk of default. Next, we turn to studying these effects.

5.2 Shocks to the World Interest Rate

Next, we measure the effects of introducing a time-varying level and volatility in the world interest rate. We do so by comparing the baseline simulation results with the simulation results for the “full model” (where all shocks are present).

We simulate and compute statistical moments from the full model in the same way as we did for the baseline model. The stochastic processes for the world interest rate (described in equations 7 – 9) take the values in the aforementioned Table 2.¹²

What are quantitative effects of introducing shocks to the world interest rate? The fourth

¹¹ Hatchondo and Martinez (2009), Hatchondo et al. (Forthcoming), and Chatterjee and Eyigungor (2012) (among others) study related models with long-duration bonds and show that having long-term debt can account for high and volatile spreads.

¹² It is important to note that there is no re-calibration across models, only the baseline economy is calibrated to match the observed debt-to-income ratio and default frequency. The only difference between the baseline and full models is the absence/presence of shocks to the world interest rate.

column in Table 3 shows that shocks to the world interest rate generate increases in the mean spread, the volatility of the spread, the default frequency; and at the same time generate a decrease in the level of debt.

The full model features roughly 170% more default risk than the baseline model with the annual default rate increasing from 2.9% to 7.9%. This increased default risk translates in higher sovereign spreads. The mean spread observed in the full model simulations is 115% higher (.28 vs .13), and the volatility of the spread is 126% higher (.52 vs .23).¹³

The sovereign borrowing optimally responds to changes in the conditions: faced with higher prices and increased uncertainty (about what is going to be the world interest rate in the future), the government engages in precautionary behavior by decreasing the average indebtedness level (the mean debt-to-income level falls from 60% to 48.7%).

In order to shade more light on the workings of how shocks to the level and to the volatility of the world interest rate we next examine how borrowing opportunities, default incentives and savings functions change.

5.2.1 Effect on Borrowing Opportunities and Policies

In Figure 2 we present the spread demanded by lenders as a function of the face value of next-period debt. The figure also shows the combination of spread levels and next-period debt chosen by the government when its initial debt level is the average level in the simulations of each case considered in the graph (i.e. baseline and full models).

Figure 2 shows that a shift in the government's choice set plays an important role in accounting for the increase in spreads implied by the incorporation of shocks to the world interest rate: Even for the same debt levels as in the baseline economy, spread levels are higher in the full model than in the baseline model. For the equilibrium debt levels in the baseline, equilibrium spread levels would be about 8,000 basis points higher in the economy with world interest rate shocks (implying that for that debt level the country will almost surely default).

Next, we turn to analyzing how stochastic volatility affects the savings policy function, $b'(b, \Theta)$, and the bond price paid in equilibrium, $q(b'(b, \Theta), \Theta)$. Figure 3 shows the behavior

¹³ Even though the relative changes in the spread statistics are significant, the absolute magnitudes are rather small: average spread increases roughly by 15 basis points, while the volatility of the spread went up by 29 basis points. These results are not entirely surprising: as explained above, sovereign default models with one-period debt have a very hard time in generating sizable and volatile spreads. See footnote 11.

of these two functions. Contrary to Figure 2 which presented a comparison across models (baseline vs. full economies), we now are focusing only on the full economy in order to see how the government’s saving decisions (and the bond price it faces) change at different levels of stochastic volatility.

The left panel of Figure 3 shows that (given the mean level of income and the mean realization of the interest rate level shock) facing increased stochastic volatility the government optimally decides to borrow less: it engages in “precautionary savings.” On the other hand, the right panel of the figure presents the bond prices faced by the government in cases of low and high stochastic volatility: the higher the volatility the worse the prices offered for its sovereign bonds.

5.2.2 Effect on Default Sets

Figure 4 plots the default sets for both the baseline and the full models. These default sets have the expected shape: for a given level of debt-to-income ratio, the country is more likely to default when it gets low realizations of income; for a given level of income, the country is more likely to default when facing higher indebtedness. We can see from this figure that the default set expands when moving from the baseline to the full model: there are more states of the world where the country will prefer to default (for a given interest rate level). This figure helps us understand the role that interest rate uncertainty plays on default incentives and explains in part why the average default frequency observed in the full model simulations is higher than in the baseline model.

5.3 Disentangling the Shocks: Intermediate Model

How important is the contribution of stochastic volatility shocks (i.e. the shocks to the volatility of the world interest rate) over and above mean volatility shocks (i.e. deviations from mean of the world interest rate)? In other words, we want to disentangle the results of the full model to study the relative contribution of u_r vs u_{σ_r} .

Table 4 presents simulations results for the three models: baseline, intermediate and full.¹⁴ We can see that, as expected, the intermediate version of our model (one in which there are

¹⁴ In the interest of avoiding unnecessary cluttering, Table 4 does not have a column with Argentine data.

interest rate level shocks but not volatility shocks, $u_{\sigma_r} = 0$) produces business cycle statistics that are in between the baseline and the full model.

From studying this table we can measure the incremental effect of volatility shocks to the world interest rate: in particular, we see that including volatility shocks increases the mean spread by 6 basis points, the volatility of the spread by 11 basis points, and the default frequency by 1.8 percentage points; while reducing the mean debt-to-income ratio by 3 percentage points. Put in different terms, volatility shocks are responsible for 39% of the increase in the mean spread, 36% percent of the increase in the volatility of the spread, 36% of the increase of the default frequency and 24% of the reduction in average debt-to-income ratio.

Figure 5 shows that the intermediate model in fact lies ‘in the middle.’ This figure presents default sets and borrowing opportunities in the three versions of the model, and confirms the basic intuition that a model with some (but not all of the) shocks to the world interest rate will exhibit larger default sets and more constrained borrowing opportunities than an otherwise identical model without any shocks to the risk-free rate.¹⁵

The take-away message from this subsection is that third order shocks, while relatively small when compared to shocks to the first and second moments, are still relevant: they explain between 24% and 39% of the variation in the statistics of interest.

5.4 Welfare Gains of Eliminating Interest Rate Uncertainty

Following the previous results it is then natural to ask: what is the welfare cost of being exposed to shocks to the world interest rate? Or equivalently, what are the welfare gains of getting rid of the uncertainty in the world interest rate? We compute these welfare gains as follows:

$$\left(\frac{v_{\text{baseline}}^0(b, \Theta)}{v_{\text{alternative}}^0(b, \Theta)} \right)^{\frac{1}{1-\gamma}} - 1 \quad (19)$$

where alternative = {full, intermediate}. Equation (19) measures the welfare gains of moving to the baseline economy. The gain is expressed as the constant proportional change in consumption that would leave a consumer indifferent between continuing living in the alternative economy (either full or intermediate) and moving to the baseline economy.

¹⁵ The stochastic modeling of the world interest rate process is simple and parsimonious, so it is reassuring that feeding an intermediate version of it into our sovereign default model produces in fact ‘intermediate’ results.

Figure 6 plots these gains as a function of the income level. The top panel of the figure shows gains attained by moving both from the full and the intermediate economies to the baseline economy (assuming zero initial debt). Both gains are positive and sizable. They are also decreasing functions of the level of income: interest rate volatility is much costlier at low levels of income (as expected). The average welfare gains of moving to the baseline economy are 1.1% (from the full model) and 0.9% (from the intermediate model).

The bottom panel of figure 6 presents the welfare gains of eliminating both shocks to the world interest rate (i.e. moving from the full to the baseline model) for two different levels of initial debt: zero and the average debt level observed in the full model (48.7% of mean annual income). The figure gives interesting insights to the welfare gains. Eliminating all the interest rate uncertainty is less valuable when the indebtedness level is high and the income is low, this is because in those states the government will likely default anyways and the value of defaulting ($v^d(\Theta)$) under no interest rate uncertainty is not dramatically higher than with uncertainty. However, for intermediate levels of income, the welfare gains are much higher (than in the zero initial debt case): it is precisely in those states where not being exposed to uncertainty makes the government able to repay and also to borrow at cheaper rates. The average welfare gain of eliminating all uncertainty about the world interest rate in this case (with initial debt equal to the mean level observed in the simulations) is equal to a 1.8% constant increase in consumption.

5.5 Impulse Response Functions

All the results presented so far have shown the quantitative relevance of both mean volatility and stochastic volatility shocks to the world interest rate. Up to now, we have been analyzing the average long-run behavior of the model and how it is affected by said shocks.¹⁶

The last set of results deals with ‘short run’ phenomena. We construct a Vector Autoregressive system to generate Impulse Response Functions (IRF) from our simulated data series. Each time period represents a quarter. Figure 7 shows the impulse responses to a one percentage point increase in the world interest rate uncertainty. Thin gray lines represent 2 standard-error bands. The responses of the world interest rate, the default probability, the sovereign spread, the trade balance-to-income ratio, and the debt-to-income ratio are expressed in percentage

¹⁶ In other words, until this section we have been studying the properties of the ‘ergodic distribution’ of the model.

points. The responses of income and consumption are expressed in percent deviations their respective means.

We interpret the IRFs as follows. In response to an unanticipated increase in the world interest rate uncertainty, the default probability increases by around 0.05% which leads to a spike in the sovereign bond spread over the world interest rate. As seen in Figure 7, the sovereign bond spread increases by around 0.10% initially and then slowly returns to its long-run mean. The government reduces its debt on impact as borrowing becomes more expensive now. As one would expect, income and consumption both fall initially and then recover gradually. At the same time, the trade balance improves on impact (which means that consumption is falling more than income on impact) and subsequently returns to its long-run mean level.

This impulse-response exercise is our model-based ‘tapering talk experiment.’ The model produces a remarkable correlation with the observed reactions in EMEs following the Fed’s initial tapering talk (in the summer of 2013) when a sharp reversal in capital flows and a spike in government bond yields occurred.

Moreover, the correlations implied by the IRFs are borne out by the data. Table C1 shows that, consistent with our IRFs, world interest rate volatility is positively correlated with sovereign spreads, the country rate, and the trade balance-to-income ratio, and negatively correlated with output, in Argentine data.¹⁷

6 Conclusions

We have introduced world interest rate uncertainty in a standard sovereign default framework à la Eaton and Gersovitz (1981). The process for the world interest rates follows the work of Fernández-Villaverde et al. (2011) and includes both mean volatility (i.e. shocks to the level of the interest rate) and stochastic volatility (i.e. shocks to the volatility of the interest rate). We measure the effects of the increased uncertainty by comparing the simulations of this model with the ones of the baseline model without a time-varying risk-free rate. We find that introducing uncertainty about the world interest rate the model produces a mean sovereign spread that is 115% larger and 126% more volatile. The model also predicts that countries default more than

¹⁷ The strong positive correlation between the world interest rate volatility and the country spread is not exclusive to Argentina: Table C2 reports positive and high correlations between these series for five Latin-American countries.

twice as frequently. Moreover, the equilibrium debt-to-income ratio is 19% lower.

The welfare gains from eliminating uncertainty about the world interest rate amount up to a 1.8% permanent increase in consumption. The model also generates Impulse Response Functions in response to an unexpected increase in world interest rate uncertainty that are consistent with the observed behavior of EMEs during the ‘tapering talk’ of 2013. Taking these two results into account, we do find quantitative support for the policy concerns (in EMEs) regarding the uncertainty about the future directions of the Fed monetary policy, and in particular about the unwinding of its quantitative easing.

Looking forward, we plan to incorporate long-duration bonds to the model. Doing so will improve the quantitative fit of the baseline economy. Furthermore, given that shocks to the world interest rate (both level and volatility ones) can be understood as a form of ‘roll-over risk’ (because they affect negatively the lender’s willingness to invest in the country), incorporating long-term bonds will allow us to study how uncertainty about the world interest rate affects the optimal maturity structure of government debt.

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A Tables

Table 1: Parameters of the Baseline Economy

Household risk aversion	γ	2	Standard value
Household's discount factor	β	0.95	Standard value
Income auto-correlation coefficient	ρ_y	0.9317	Argentina's GDP
Std. dev. of income innovations	σ_ε	0.037	Argentina's GDP
Mean int'l risk-free rate	\bar{r}	0.01	Average world interest rate
Prob. of re-entry	μ	0.0385	Chatterjee and Eyigungor (2012)
Default cost parameter	d_0	-1.45	Calibrated to fit targets
Default cost parameter	d_1	1.50	Calibrated to fit targets

Table 2: Parametrization of the World Interest Rate Process

Autocorrelation risk-free rate	ρ_r	0.95
Mean volatility of int'l risk-free rate	$\bar{\sigma}_r$	-6.959
Autocorrelation interest vol. shock	ρ_{σ_r}	0.94
Stochastic vol. of int'l risk-free rate	η_r	0.1466

Note: The calibration of the stochastic process governing the world interest rate and its time-varying volatility are adapted from Fernández-Villaverde et al. (2011). This parameter values do not affect the performance of the baseline economy (as this economy has neither time-varying level of the interest rate nor time-varying volatility of the interest rate).

Table 3: Business Cycle Statistics

	Data	Baseline Model	Full Model
$sd(c)/sd(y)$	1.59	1.68	1.85
$corr(c, y)$	0.72	0.93	0.88
$corr(tb/y, y)$	-0.64	-0.67	-0.59
$E(R_s)$ (in %)	7.44	0.13	0.28
$sd(R_s)$ (in %)	2.51	0.23	0.52
$corr(R_s, y)$	-0.62	-0.57	-0.55
$E(b/y)$ (in %)	59.9	60.0	48.7
Default frequency (in %)	3.0	2.9	7.9

Note: The mean and the standard deviation of a variable x are denoted by $E(x)$ and $sd(x)$, respectively. The coefficient of correlation between two variables x and z is denoted as $corr(x, z)$. All variables are logged (except those that are ratios) and then de-trended using the Hodrick-Prescott filter, with a smoothing parameter of 1,600. We report deviations from the trend.

R_s stands for sovereign bond spread. The data for sovereign spreads is taken from J.P. Morgan's EMBI+, which represents the difference in yields between an Argentine bond and a US bond of similar maturity.

Only Baseline model is calibrated to match $E(b/y)$ and default frequency. Parameters are kept unchanged across models (except for those that turn on/off shocks to the world interest rate).

Table 4: Simulation Results: Baseline, Intermediate and Full Models

	Baseline Model	Intermediate Model	Full Model
$sd(c)/sd(y)$	1.68	1.80	1.85
$corr(c, y)$	0.93	0.89	0.88
$corr(tb/y, y)$	-0.67	-0.61	-0.59
$E(R_s)$ (in %)	0.13	0.22	0.28
$sd(R_s)$ (in %)	0.23	0.41	0.52
$corr(R_s, y)$	-0.57	-0.55	-0.55
$E(b/y)$ (in %)	60.0	51.4	48.7
Default frequency (in %)	2.9	6.13	7.9

Note: The mean and the standard deviation of a variable x are denoted by $E(x)$ and $sd(x)$, respectively. The coefficient of correlation between two variables x and z is denoted as $corr(x, z)$. All variables are logged (except those that are ratios) and then de-trended using the Hodrick-Prescott filter, with a smoothing parameter of 1,600. We report deviations from the trend.

R_s stands for sovereign bond spread. The data for sovereign spreads is taken from J.P. Morgan's EMBI+, which represents the difference in yields between an Argentine bond and a US bond of similar maturity.

Only Baseline model is calibrated to match $E(b/y)$ and default frequency. Parameters are kept unchanged across models (except for those that turn on/off shocks to the world interest rate).

B Figures

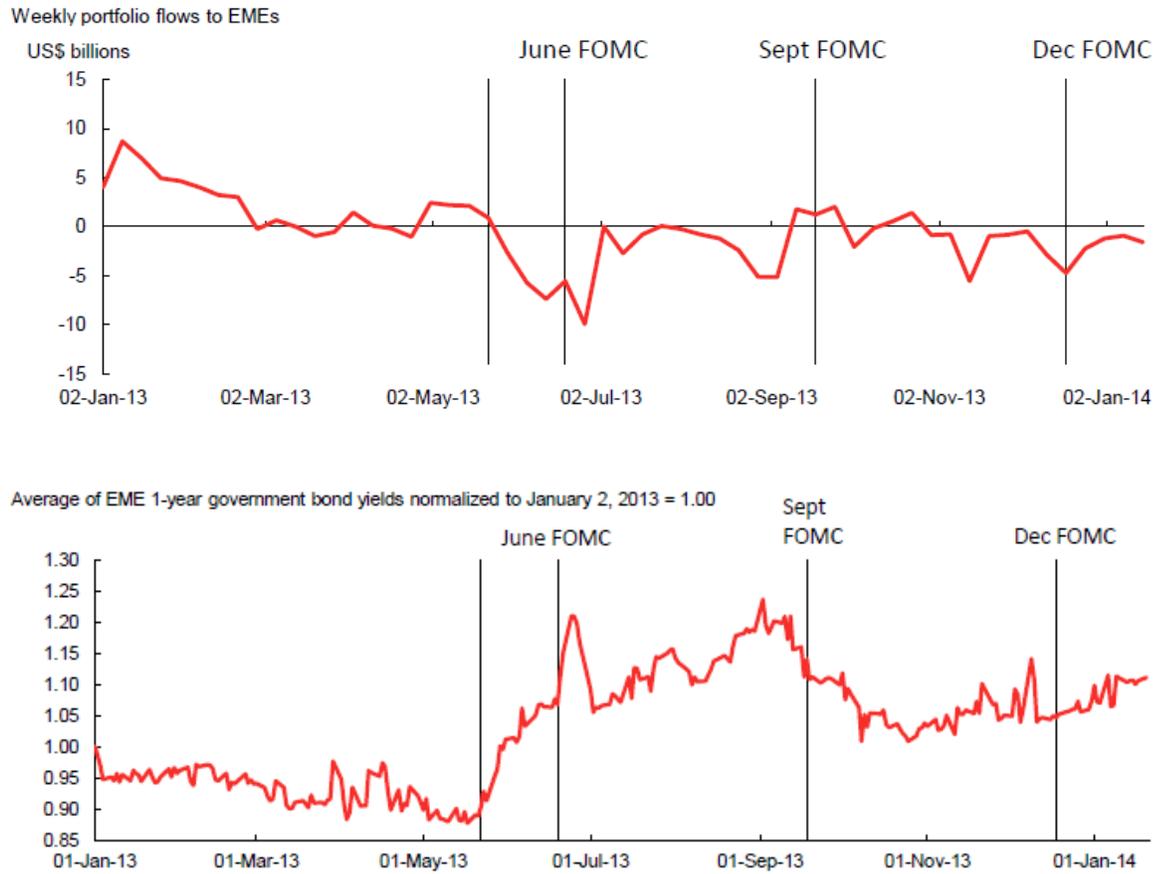


Figure 1: Capital flows reversals and sovereign bond yield increases on tapering announcement. The top (bottom) panel shows data on capital flows (sovereign bond yields).

Note: Calculations based on 19 EMEs (Brazil, China, Chile, Columbia, Czech Republic, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Russia, Taiwan, Thailand, Turkey, and South Africa).

Data sources: Bloomberg, Bank of Canada, and EPFR.

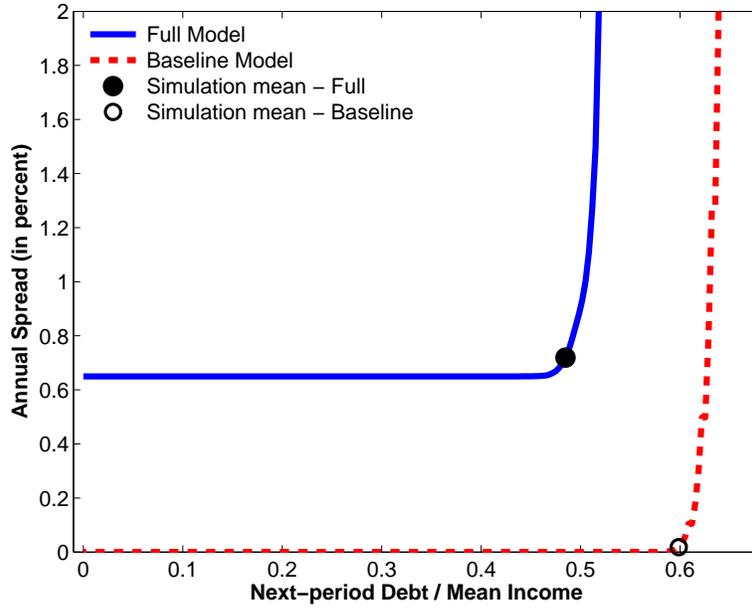


Figure 2: Menu of combinations of spreads and next-period debt levels from which the government can choose. The solid blue (dashed red) line corresponds to the full (baseline) model.

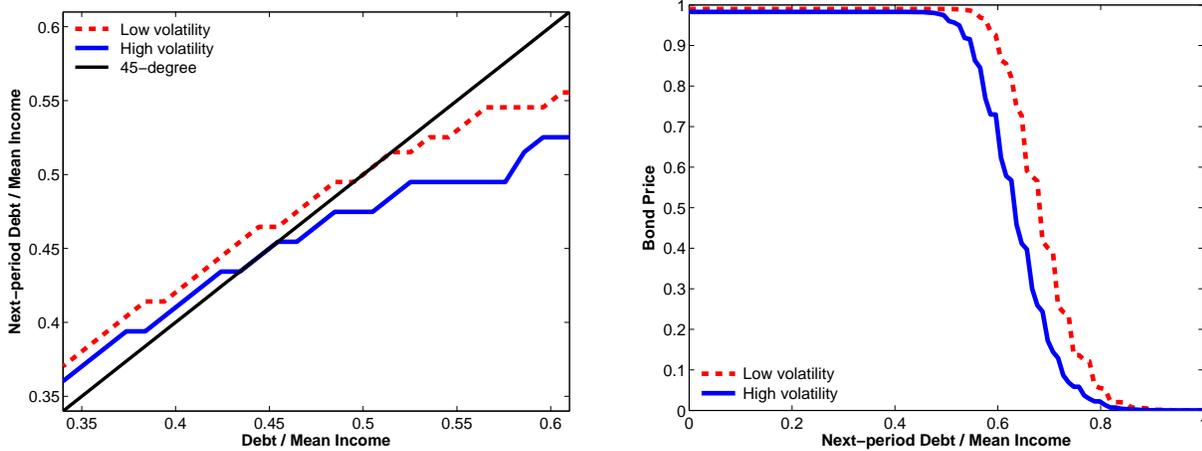


Figure 3: Savings and Bond Price functions in the Full Model. The left panel corresponds to the equilibrium savings function, $b'(b, \Theta)$. The right panel corresponds to equilibrium bond price function, $q(b'(b, \Theta), \Theta)$. The solid blue (dashed red) line corresponds to high (low) volatility, $\bar{\sigma}_r = -5.24$ ($\bar{\sigma}_r = -8.68$). Both panels assume the average income level and the average interest rate level shock.

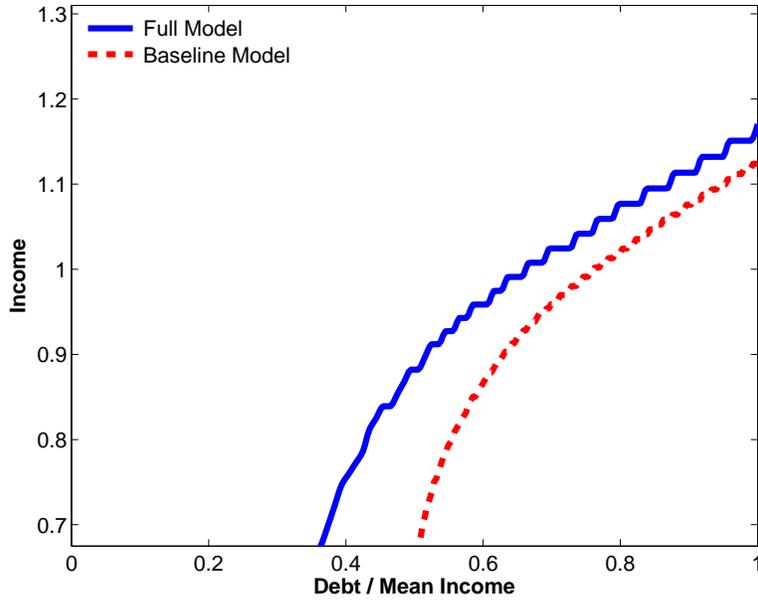


Figure 4: Default sets. The solid blue (dashed red) line corresponds to the full (baseline) model. Each line is the respective default set contour: country defaults *south* of the line. The figure assumes the average mean and stochastic volatility for the world interest rate.

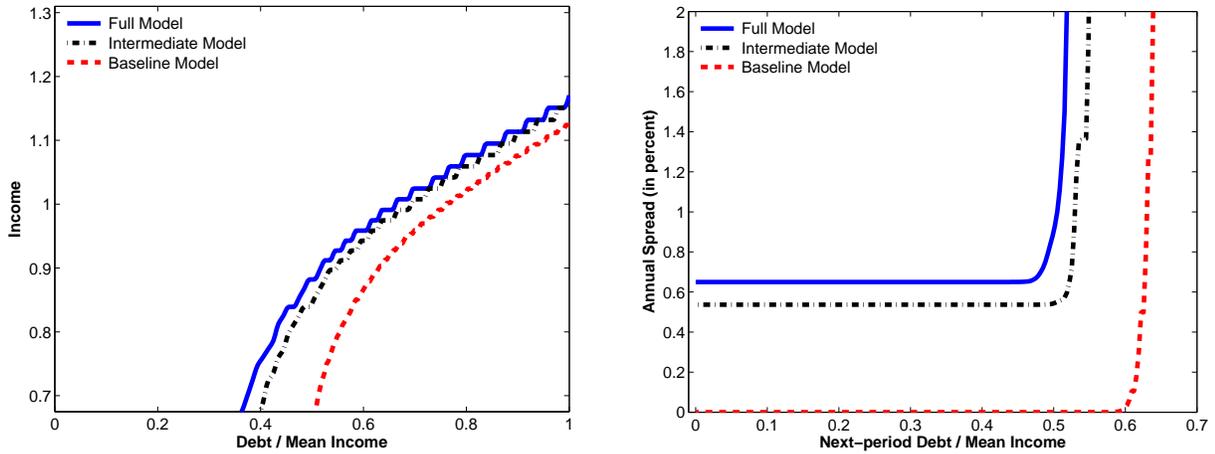


Figure 5: Default sets and borrowing opportunities: baseline, intermediate and full models.

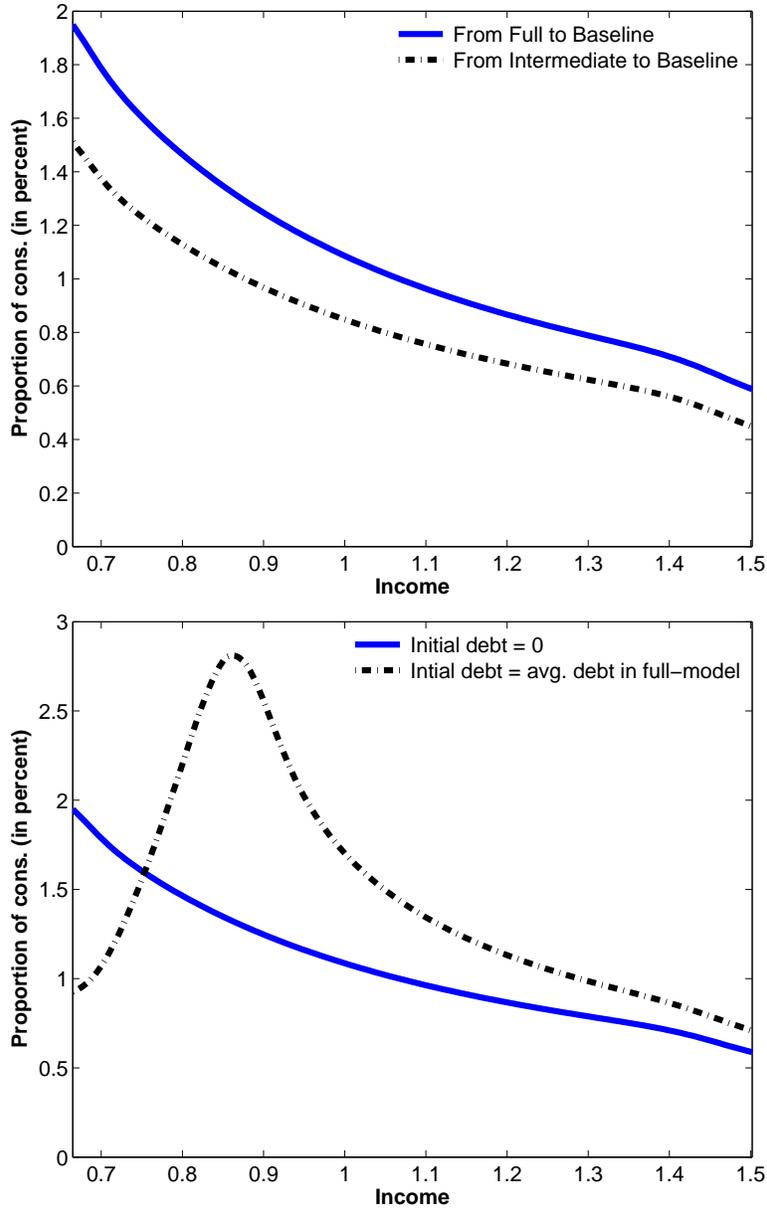


Figure 6: Welfare gains of moving to the baseline economy. The top panel assumes a zero initial debt and plots the gains of moving to the baseline economy from the full model (solid line) and from the intermediate model (dashed line). The bottom panel plots only gains of moving from the full to the baseline economy, for two levels of initial debt: zero (solid line) and the avg. debt level in the simulations of the full-model (dashed line).

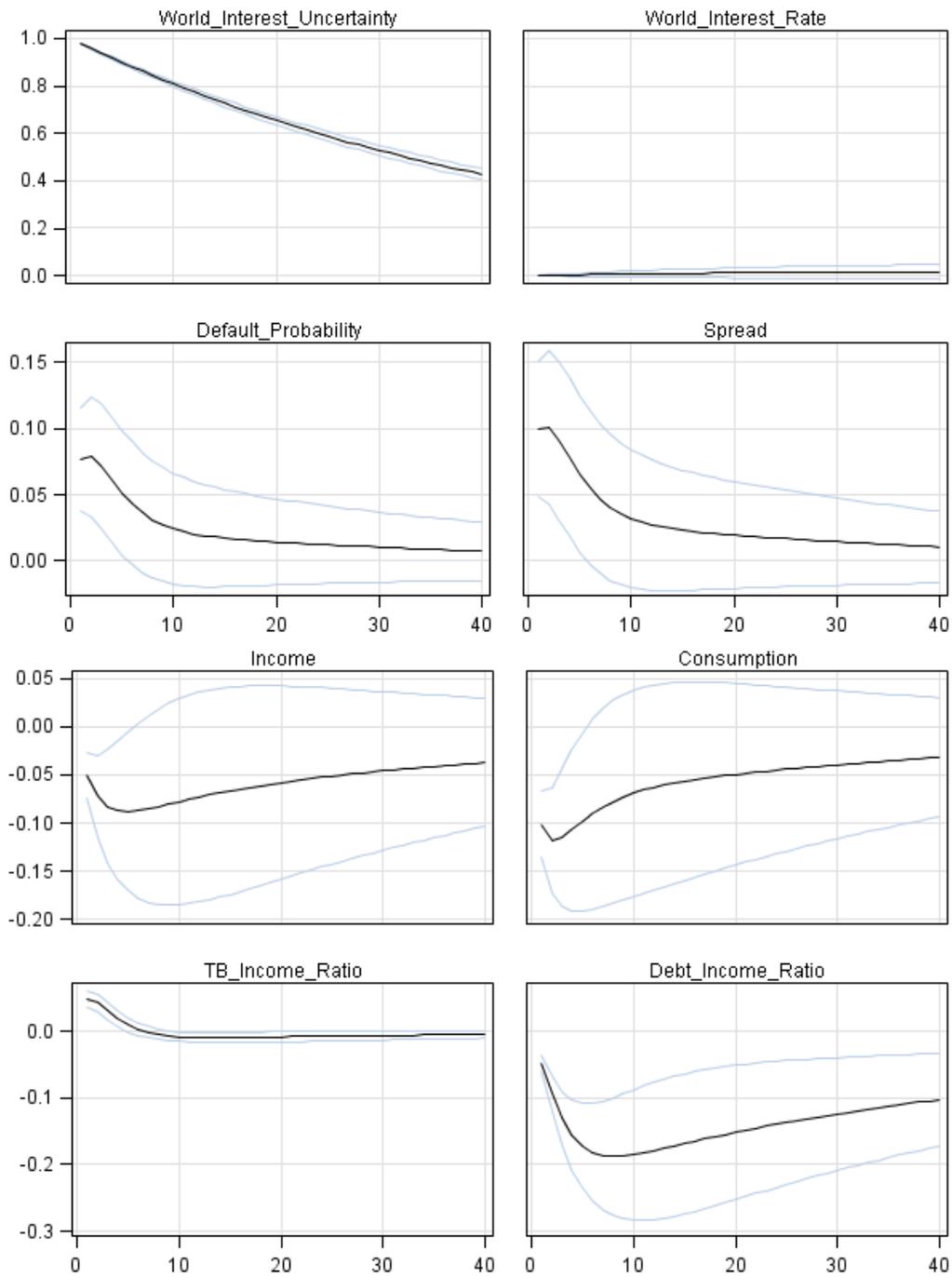


Figure 7: Impulse Response Functions. The panels show responses to a 1 percentage point increase the world interest rate uncertainty. Thin gray lines represent 2 standard-error bands.

C Supplementary Tables

Table C1: Correlation coefficients among key macro variables for Argentina.

	Int. volatility	Country rate	Spread	GDP	$\frac{TB}{GDP}$
Int. volatility	1.00				
Country rate	0.28	1.00			
Spread	0.31	0.99	1.00		
GDP	-0.22	-0.77	-0.72	1.00	
$\frac{TB}{GDP}$	0.52	0.78	0.81	-0.64	1.00

Note: World interest rate volatility is computed as the standard deviation of U.S. 3-month T-bill real interest rate growth series. GDP is seasonally adjusted and de-trended using a log-linear trend. Time-period: 1994.Q4–2012.Q4. Data sources: NIPA, IFS, J.P. Morgan EMBI+, FRB St. Louis.

Table C2: Correlations btw. world interest rate volatility and country spreads in Latin-America.

	Int. volatility	Argentina	Brazil	Colombia	Ecuador	Uruguay
Int. volatility	1.00					
Argentina	0.19	1.00				
Brazil	0.41	0.75	1.00			
Colombia	0.48	0.78	0.94	1.00		
Ecuador	0.57	0.18	0.35	0.52	1.00	
Uruguay	0.46	0.81	0.85	0.84	0.44	1.00

Note: World interest rate volatility is computed by taking the standard deviation of U.S. 3-month T-bill real interest rate growth series. Spread is the difference between real yields on country dollar-denominated bonds and U.S. treasuries.

Time-period: 2001.Q2–2012.Q4. Data sources: IFS, J.P. Morgan EMBI+, FRB St. Louis.